学校编码：10384　　　　　　　　　　　　　　　　　　　　　　密级______

学号：25320111151729

厦门大学

硕　士　学　位　论　文

结构振动超收敛等几何分析方法

Ultra-Accurate Isogeometric Analysis of Structural Vibrations

刘伟

指导教师姓名：　王东东　教授

专　业　名　称：　建筑与土木工程

论文提交日期：　2014 年 4 月

论文答辩日期：　2014 年 5 月

2014 年 5 月

厦门大学学位论文原创性声明

本人呈交的学位论文是本人在导师指导下,独立完成的研究成果。本人在论文写作中参考其他个人或集体已经发表的研究成果，均在文中以适当方式明确标明，并符合法律规范和《厦门大学研究生学术活动规范（试行）》。

另外，该学位论文为（ ）课题（组）的研究成果，获得（ ）课题（组）经费或实验室的资助，在（ ）实验室完成。（请在以上括号内填写课题或课题组负责人或实验室名称，未有此项声明内容的，可以不作特别声明。）

声明人（签名）：

年 月 日

厦门大学学位论文著作权使用声明

本人同意厦门大学根据《中华人民共和国学位条例暂行实施办法》等规定保留和使用此学位论文，并向主管部门或其指定机构送交学位论文（包括纸质版和电子版），允许学位论文进入厦门大学图书馆及其数据库被查阅、借阅。本人同意厦门大学将学位论文加入全国博士、硕士学位论文共建单位数据库进行检索，将学位论文的标题和摘要汇编出版，采用影印、缩印或者其它方式合理复制学位论文。

本学位论文属于：

（　　）1.经厦门大学保密委员会审查核定的保密学位论文，于　　年　　月　　日解密，解密后适用上述授权。

（ √ ）2.不保密，适用上述授权。

（请在以上相应括号内打“√”或填上相应内容。保密学位论文应是已经厦门大学保密委员会审定过的学位论文，未经厦门大学保密委员会审定的学位论文均为公开学位论文。此声明栏不填写的，默认为公开学位论文，均适用上述授权。）

声明人（签名）：

年　　月　　日

摘要

等几何分析方法采用非均匀有理 B 样条形函数进行结构模型描述和计算分析，可以消除几何建模误差和传统有限元质量矩阵中负元素的影响，有效提高结构振动频率的计算精度。本文在此基础上提出了结构振动分析的超收敛等几何分析方法，其基本思想是通过构造高阶质量矩阵来提高结构振动频率的计算精度。文中针对一维杆结构、二维膜结构和薄板结构，提出了一种新型两步法来构造结构振动分析的高阶质量矩阵，即首先构造一个与一致质量矩阵在频率上具有相同收敛阶次的缩减带宽质量矩阵，然后通过缩减带宽质量矩阵与一致质量矩阵的线性优化组合构建高阶质量矩阵。另一方面，对于欧拉梁振动问题，相应的高阶质量矩阵可以通过优化缩减带宽质量矩阵直接得到。

对于杆结构振动问题，采用高阶质量矩阵的超收敛等几何分析方法相对于采用一致质量矩阵振动分析方法得到的频率收敛阶次提高了 2 阶。针对二次和三次基函数的情况，频率收敛阶次分别由 4 次和 6 次收敛达到 6 次和 8 次收敛。对于欧拉梁问题，针对二次和三次基函数的情况，频率收敛率可以分别由原来的 2 次和 4 次提高至 4 次和 6 次收敛。然后进一步通过张量积运算方法，采用一维缩减带宽质量矩阵建立了二维膜结构和薄板结构的缩减带宽质量矩阵，并与一致质量矩阵采用线性优化组合得到相应的高阶质量矩阵，其频率精度相对于传统一致质量矩阵收敛率均提高了 2 阶。文中通过一系列典型算例全面深入地验证了所提出的结构振动超收敛等几何分析方法。

关键词：等几何分析　结构振动　缩减带宽质量矩阵　高阶质量矩阵　频率误差　超收敛方法

Abstract

The isogeometric analysis employs the non-uniform rational B-splines as the basis functions simultaneously for geometric description and finite element analysis. The method is capable of eliminating the geometric discretization errors and the effect of negative elements for the mass matrix, as provides a very favorable way for structural vibration analysis. In this thesis an ultra-accurate isogeometric analysis of structural vibrations is proposed. The essential component for the proposed method is to construct novel higher order mass matrices for the vibration analysis. More specifically, for the 1D rod problems, 2D membrane problems and thin plate problems, the higher order mass matrices are formulated using a new two-step method. Firstly based upon the standard consistent mass matrix a special reduced bandwidth mass matrix is designed. This reduced bandwidth mass matrix meets the requirement of mass conservation while also preserves the same order of frequency accuracy as the corresponding consistent mass matrix. Subsequently a mixed mass matrix is formulated through a linear combination of the reduced bandwidth mass matrix and the consistent mass matrix. The desired higher order mass matrix is then deduced from the mixed mass matrix by optimizing the linear combination parameter to achieve the most favorable order of accuracy. Nonetheless, for the Euler beam problems, it turns out that the desired higher order mass matrices can be directly established through optimally reducing the bandwidth of consistent mass matrix.

For the 1D rod vibration problems, an elevation of two orders of accuracy for the vibration frequencies is observed for the proposed ultra-accurate isogeometric method compared with the standard isogeometric analysis with the consistent mass matrices. It is shown that for 1D rod problems, with regard to the vibration frequency the proposed higher order mass matrices have 6th and 8th orders of accuracy in contrast to the 4th and 6th orders of accuracy associated with the quadratic and cubic consistent mass matrices. For Euler beam problems, the proposed higher order mass matrices have 4th and 6th orders of accuracy, while they are 2th and 4th orders of accuracy for the quadratic and cubic consistent mass matrix formulations. A generalization to 2D higher order mass matrices are further realized by the tensor product operation on the one dimensional reduced bandwidth and consistent mass

matrices. It is proved that the 2D higher order mass matrices also enable an ultra-accurate analysis for membranes and thin plates by improving two orders of frequency accuracy in comparison with the vibration analysis with consistent mass matrices. A series of benchmark examples congruously demonstrate that the proposed higher order mass matrices are capable of achieving the theoretically derived optimal accuracy orders for structural vibration analysis.

Key Words: Isogeometric Analysis; Structural Vibration; Reduced Bandwidth Mass Matrix; Higher Order Mass Matrix; Frequency Error; Ultra-Accurate Formulation

目录

摘要 i

Abstract iii

目录 v

Contents ix

第一章 绪论 1

1.1 引言 1

1.2 等几何分析的研究现状 2

1.3 论文的选题背景 4

1.4 本文的主要内容 6

第二章 等几何结构振动分析 7

2.1 B 样条与 NURBS 基函数 7

2.2 等几何结构振动分析 11

2.1.1 模型问题 12

2.1.2 等参变换 13

2.1.3 等几何离散 18

2.3 本章小结 29

第三章 一维杆振动问题的超收敛等几何分析方法 31

3.1 一维杆振动问题的高阶质量矩阵 31

3.1.1 二次基函数 31

3.1.2 三次基函数 35

3.2 一维杆问题算例 38

3.3 本章小结 44

第四章 二维膜振动问题的超收敛等几何分析方法......45

4.1 二维膜振动问题的高阶质量矩阵......45

4.2 二维方形膜问题算例......52

4.2.1 周期性基函数分析......53

4.2.2 开放型节点矢量基函数分析......56

4.3 二维圆形膜问题算例......58

4.4 本章小结......62

第五章 欧拉梁振动问题的超收敛等几何分析方法......63

5.1 欧拉梁振动问题的高阶质量矩阵......63

5.1.1 二次基函数......63

5.1.2 三次基函数......65

5.2 欧拉梁问题算例......68

5.3 本章小结......75

第六章 薄板振动问题的超收敛等几何分析方法......77

6.1 薄板振动问题的高阶质量矩阵......77

6.1.1 二次基函数......77

6.1.2 三次基函数......83

6.2 方形薄板问题算例......95

6.2.1 二次周期性基函数分析......95

6.2.2 二次开放型节点矢量基函数分析......98

6.2.3 三次周期性基函数分析......100

6.2.4 三次开放型节点矢量基函数分析......103

6.3 长方形薄板问题算例......105

6.3.1 二次周期性基函数分析......106

6.3.2 二次开放型节点矢量基函数分析......112

6.3.3 三次周期性基函数分析......117

6.3.4 三次开放型节点矢量基函数分析......122

6.4 圆形薄板问题算例......127

6.4.1 二次开放型节点矢量基函数分析......128
6.4.2 三次开放型节点矢量基函数分析......130
6.5 本章小结......132
第七章 结论与展望......133
7.1 结论......133
7.2 展望......134
参 考 文 献......135
致 谢......145
作者攻读硕士学位期间撰写的论文......147

Contents

Abstract.. i

Contents v

Chapter 1 Introduction...1

1.1 Introduction...1

1.2 Overview of isogeometric analysis...2

1.3 Motivation...4

1.4 Objective and scope 6

Chapter 2 Isogeometric Analysis of Structural Vibration 7

2.1 B-spline and NURBS basis functions 7

2.2 Isogeometric analysis of structural vibration...................................11

2.1.1 Model problem...12

2.1.2 Isoparametric transformation...13

2.1.3 Discretion of isogeometric analysis 18

2.3 Concluding remarks 29

Chapter 3 Ultra-Accurate Isogeometric Analysis of 1D Rod Vibration 31

3.1 Higher order mass matrices for 1D rod vibration 31

3.1.1 Quadratic basis function 31

3.1.2 Cubic basis function...35

3.2 Numerical examples for 1D rod problem..38

3.3 Concluding remarks 44

Chapter 4 Ultra-Accurate Isogeometric Analysis of 2D Membrane Vibration 45

4.1 Higher order mass matrices for 2D membrane vibration................45

4.2 Numerical examples for 2D square membrane problem................52

4.2.1 Analysis with periodic basis function 53

4.2.2 Analysis with open knot basis function 56

4.3 Numerical examples for 2D circle membrane problem............................58

4.4 Concluding remarks 62

Chapter 5 Ultra-Accurate Isogeometric Analysis of Euler Beam Vibration 63

5.1 Higher order mass matrices for Euler beam vibration 63

5.1.1 Quadratic basis function 63

5.1.2 Cubic basis function...65

5.2 Numerical examples for Euler baem problem...68

5.3 Concluding remarks 75

Chapter 6 Ultra-Accurate Isogeometric Analysis of Thin Plate Vibration 77

6.1 Higher order mass matrices for thin plate vibration...............................77

6.1.1 Quadratic basis function 77

6.1.2 Cubic basis function...83

6.2 Numerical examples for square thin plate problem................................95

6.2.1 Analysis with periodic quadratic basis function 95

6.2.2 Analysis with open knot quadratic basis function 98

6.2.3 Analysis with periodic cubic basis function......................................100

6.2.4 Analysis with open knot cubic basis function....................................103

6.3 Numerical examples for rectangle thin plate problem 105

6.3.1 Analysis with periodic quadratic basis function 106

6.3.2 Analysis with open knot quadratic basis function 112

6.3.3 Analysis with periodic cubic basis function......................................117

6.3.4 Analysis with open knot cubic basis function....................................122

6.4 Numerical examples for circle thin plate problem.................................127

6.4.1 Analysis with open knot quadratic basis function 128

6.4.2 Analysis with open knot cubic basis function....................................130

6.5 Concluding remarks 132

Chapter 7 Conclusions...133